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§2

38% : D ———
26% : — D ———
At most 5% : ~~L~~D ———

So, at least 59% ranks D ahead of L.

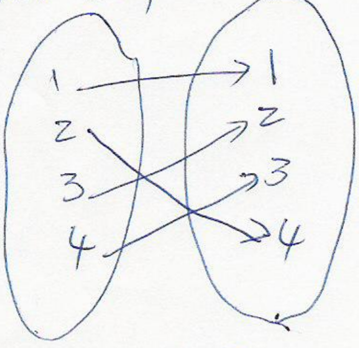
§4

problem (τ, a_0) → Criterion F → allocation $a = (a_1, \ldots, a_n)$

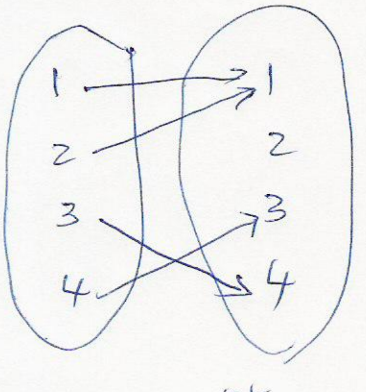
Ex. $a_0 = 2$
$\tau = (\tau_A, \tau_B, \tau_C, \tau_D, \tau_E)$
$\tau_A = (5, 2, 10, 0, 0)$ etc.

- when $a_0 = 1$, we have $F(\tau, 1) = \underline{\{(0, 0, 0, 0, 1)\}}$.

- Permutation (bijection 全単射)



permutation



Not a permutation

- Similarly, $\tau \circ \pi = \underline{(\tau_{\pi(1)}, \ldots, \tau_{\pi(n)})}$.

- $P \overset{\text{only if}}{\Rightarrow} Q$ P only if Q
  $P \Leftarrow Q$ P if Q
  $P \Leftrightarrow Q$ P iff (if and only if) Q

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- $(a_1, a_2, a_3) \in F((\tau_1, \tau_2, \tau_3), a_0)$

  $\Leftrightarrow (a_2, a_1, a_3) \in F((\tau_2, \tau_1, \tau_3), a_0)$

- Proof

  (⇒). Assume (*)[仮定せよ]. Show (**)[示す].

  $a \in F(\tau, a_0) \overset{*}{\Rightarrow} a \circ \pi \in F(\tau, a_0) \circ \pi = F(\tau \circ \pi, a_0)$.

  (⇐). $a \circ \pi \in F(\tau \circ \pi, a_0)$

  $\overset{*}{\Rightarrow} a \circ \pi \in F(\tau, a_0) \circ \pi$

  $\Rightarrow a \circ \pi = a' \circ \pi$ for some $a' \in F(\tau, a_0)$

  $\Rightarrow a = a \circ \pi \circ \pi^{-1} = a' \circ \pi \circ \pi^{-1} = a' \in F(\tau, a_0)$

  [π^{-1} is the inverse[逆写像] of π.]

  $\Rightarrow a \in F(\tau, a_0)$

- $F((\tau, \tau'), 2) = \underline{\{(1, 1)\}}$.

- Pairwise consistency

  F is pairwise consistent if for every finite $I \subseteq N$, every problem (τ, a_0) on I, and every $\{i, j\} \subset I$,

  $a \in F(\tau, a_0) \Rightarrow \underline{(a_i, a_j)} \in F((\tau_i, \tau_j), a_i + a_j)$,

  and

  $a \in F(\tau, a_0)$ and $(b_i, b_j) \in F((\tau_i, \tau_j), a_i + a_j)$

  $\Rightarrow \underline{(a_1, \ldots, a_{i-1}, \underset{i\text{th}}{b_i}, a_{i+1}, \ldots, a_{j-1}, \underset{j\text{th}}{b_j}, a_{j+1}, \ldots, a_n)} \in F(\tau, a_0)$

  Can be written $(b_i, b_j, a_{-\{i,j\}}) \in F(\tau, a_0)$.

- $F((\tau_1, \tau_2, \tau_3), 2) = \{(0, 1, 1)\} \xrightarrow{\text{Pairwise consist.}} \underline{(0, 1) \in F((\tau_1, \tau_2), 1)}$

- The original UNOS formula violates consistency:

  Suppose F is consistent. Then $(a_A, \ldots, a_E) = (0, 0, 0, 1, 1) \in F(\tau, 2)$ implies $(a_A, a_D) = (0, 1) \in F((\tau_A, \tau_D), 1)$. (1)

  On the other hand, $(a_A, \ldots, a_D) = (1, 0, 0, 0) \in F((\tau_A, \ldots, \tau_D), 1)$ and (1) imply $(0, 0, 0, 1) \in F((\tau_A, \ldots, \tau_D), 1)$, contradiction[矛盾].

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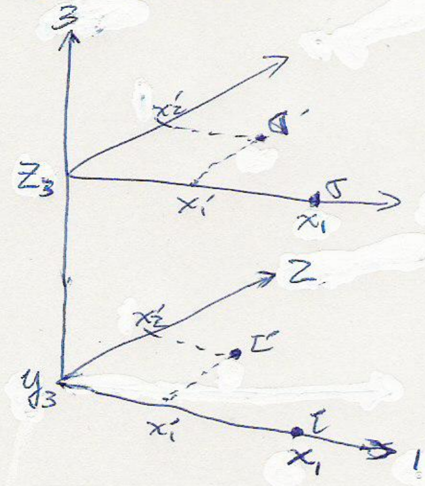
- Priority Method, Example
  giving them either
  - to A and B

  or - to A and C.

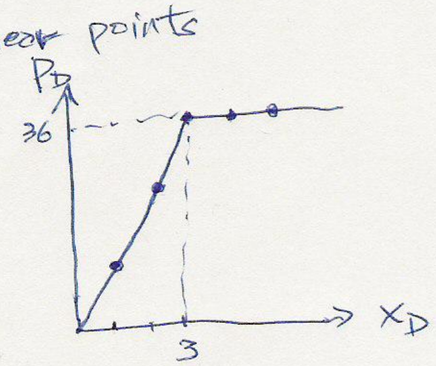
  $F((A,B,C,D),2) = \{(1,1,0,0),(1,0,1,0)\}$.

§5

- $zPz' \Leftrightarrow \sigma P \sigma'$



- Nonlinear points



§6 • The Borda score of $B = \underline{13\times 0 + 10\times 1 + 6\times 2 + 12\times 2 + 18\times 1} = 66$

$C =$ $= 62$

∴ The Borda ranking is $\underline{BCA}$.

- Vote matrix

	A	B	C	Borda Score	
A	0	23	29	52	$v_{AB} + v_{AC}$
B	37	0	$\underline{29}$	66	$v_{BA} + v_{BC}$
C	31	$\underline{31}$	0	$\underline{62}$	$v_{CA} + v_{CB}$

Consider ABC

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- $$\begin{tikzcd}
 & C \arrow[dl, "31"'] \arrow[dr, "31"] & \\
B & & A
\end{tikzcd}$$

- Majority alternative 

- B has priority over $\underline{A}$: $\underline{v_{BA}=37}$ voters support this.

- The Condorcet score of CBA is
  $v_{CB}+v_{BA}+v_{CA}=31+37+31=99$

- Voting paradox (Table 2.7)

$$\begin{tikzcd}
 & A \arrow[dr, "8 \text{ votes}"] & \\
C \arrow[ur, "7"] & & B \arrow[ll, "11"]
\end{tikzcd}$$

- A B C   A C B
  B A C   B C A
  C A B   C B A

- IIA

profile 1 $\left\{\begin{array}{l} abc\ldots \\ bac\ldots \\ bac\ldots \\ bac\ldots \end{array}\right.$ profile 2 $\left\{\begin{array}{l} ac\ldots b \\ bac\ldots \\ bac\ldots \\ bac\ldots \end{array}\right.$

Social : $\underbrace{ba}c\ldots$ $\xRightarrow{\text{IIA}}$ Social : $\ldots \underbrace{b \ldots a} \ldots$